INGENIERIA INDUSTRIAL
UNIVERSIDAD DE CHILE

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"Stochastic Forestry Planning Problem Using Progressive Hedging"

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INFORMS
Nashville 2016

ISCI INSTITUTE COMPLEX ENGINEERING SYSTEMS

Agenda

I. Background
II. Motivation
III. Case study
- Uncertainty
- Solution Approaches
- Results and Discussion

IV. General Conclusions

Background & Motivation

Chile

- ~3% of GDP
- 2nd economic activity

EE.UU.

- 1st consumer
- 2nd producer

Background & Motivation

Deterministic Models

- Great results
- But limited

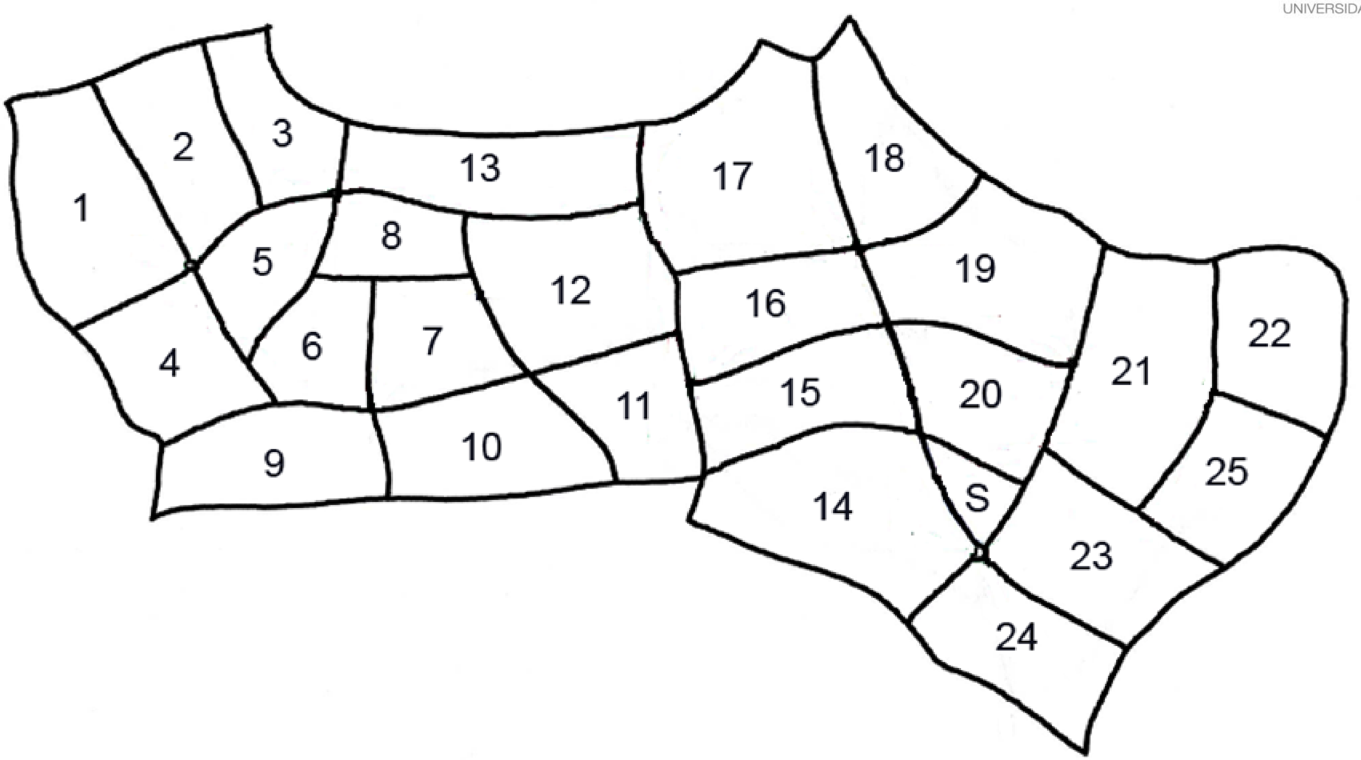
Stochastic Models

- Two Stages
- Multi-stage

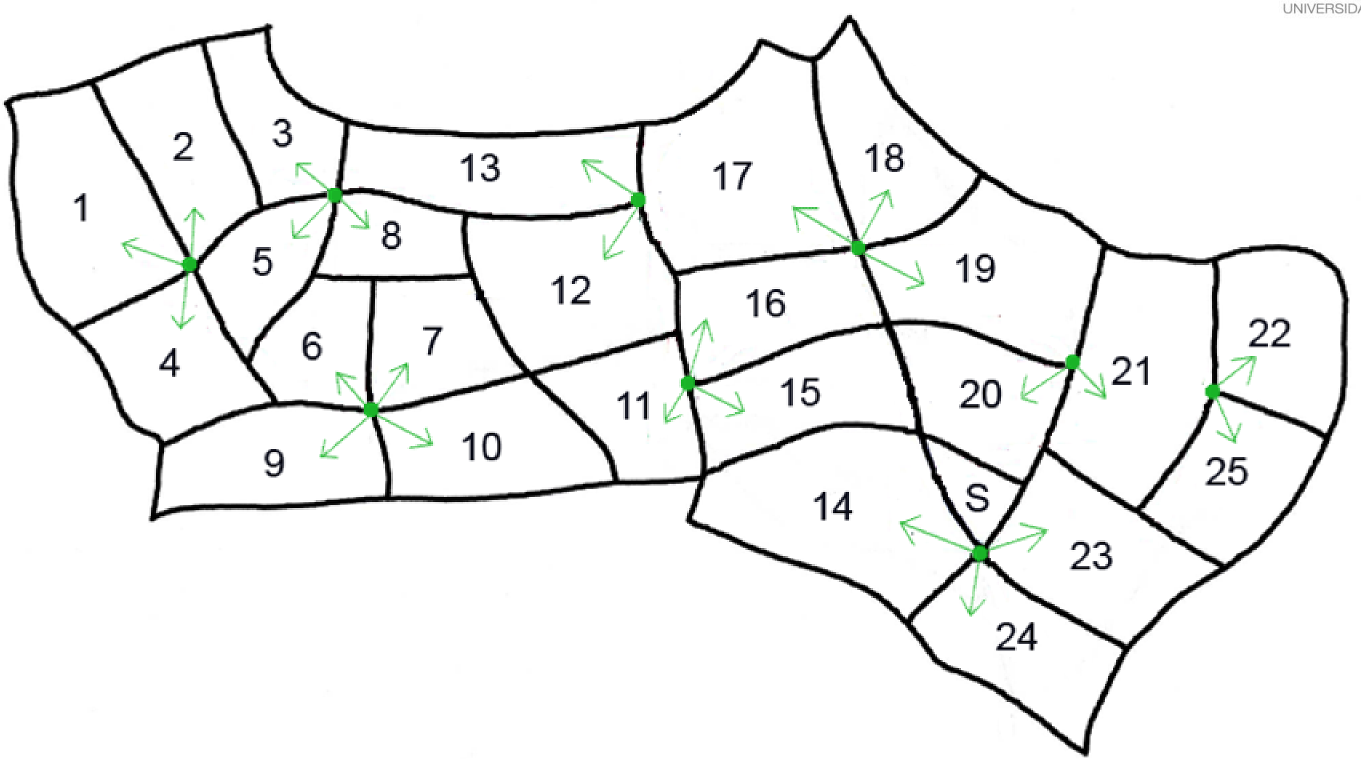
Case Study I
Millalemu Chilean Forests

“Stochastic Forestry Planning Problem using Progressive Hedging”, Pais and Weintraub (2016)

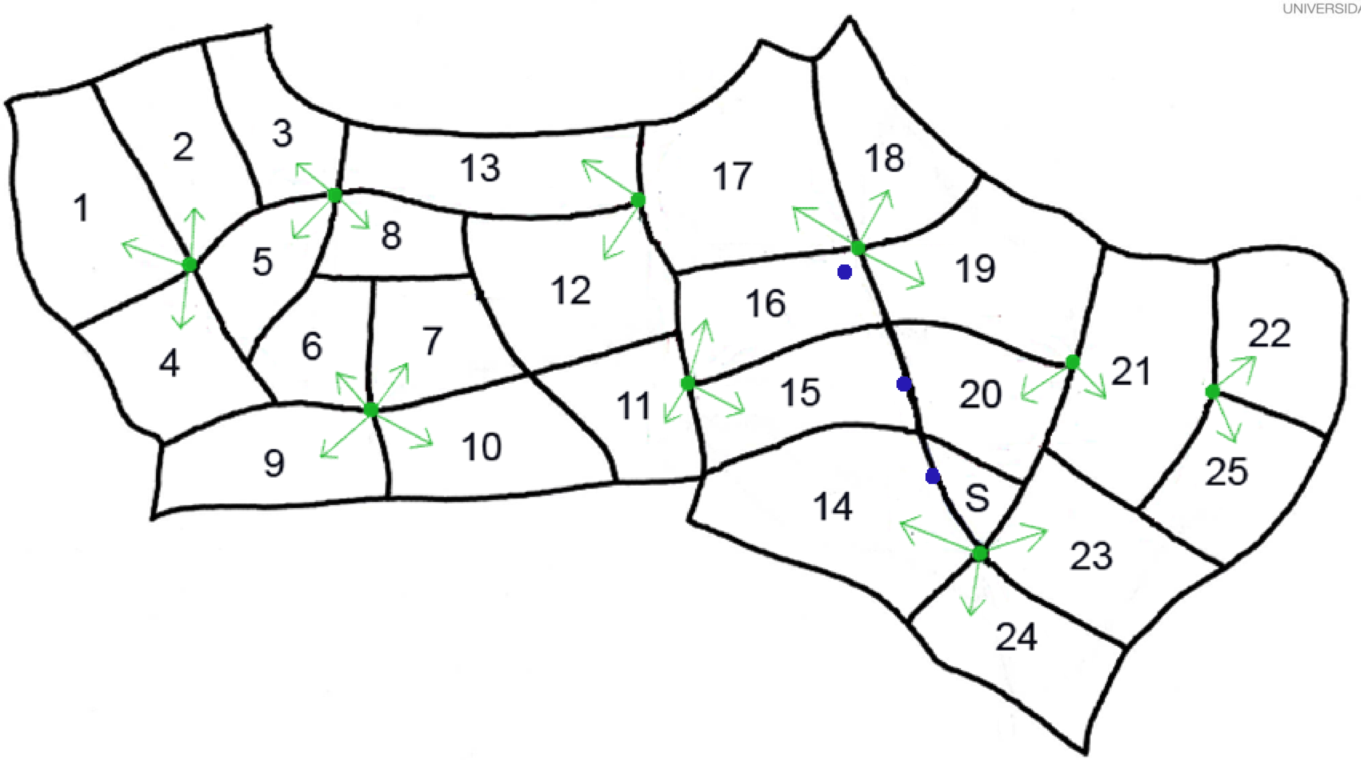
Case Study I



Case Study I



Case Study I



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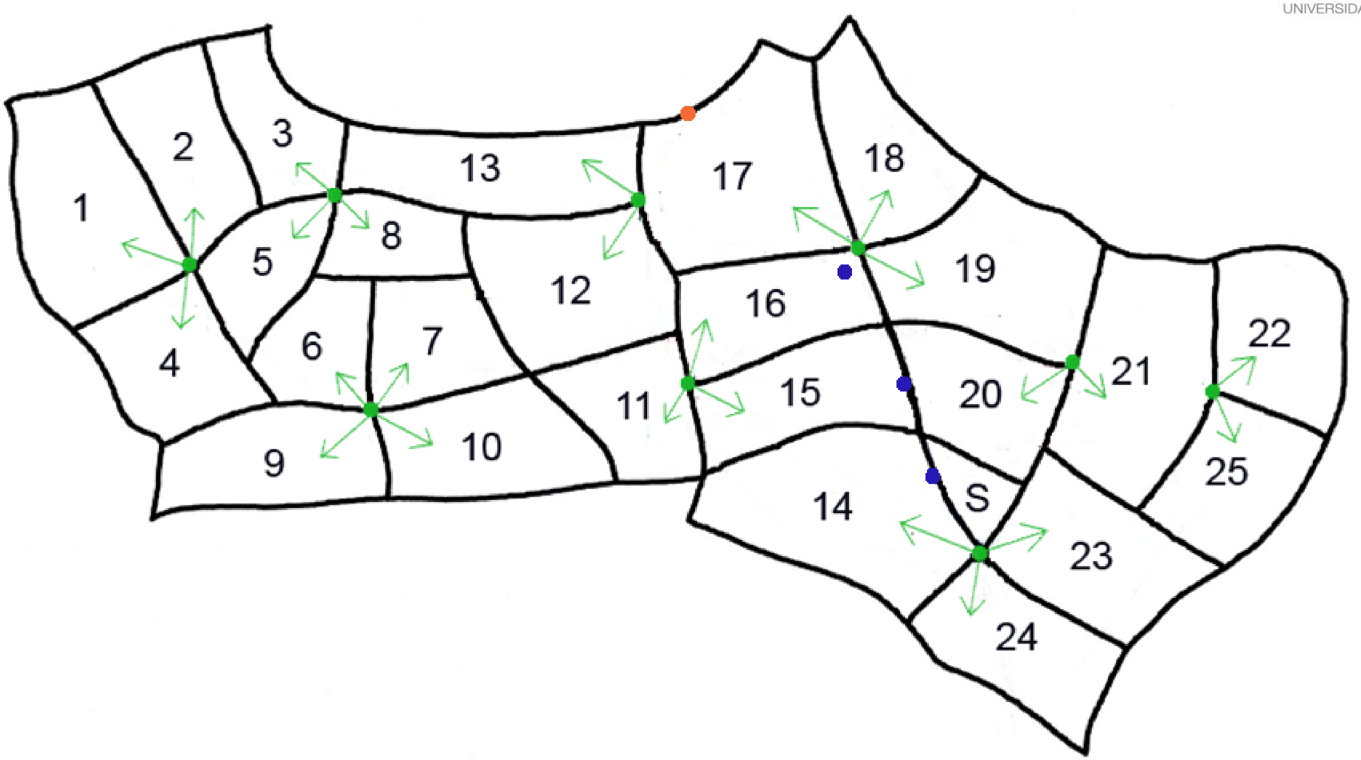
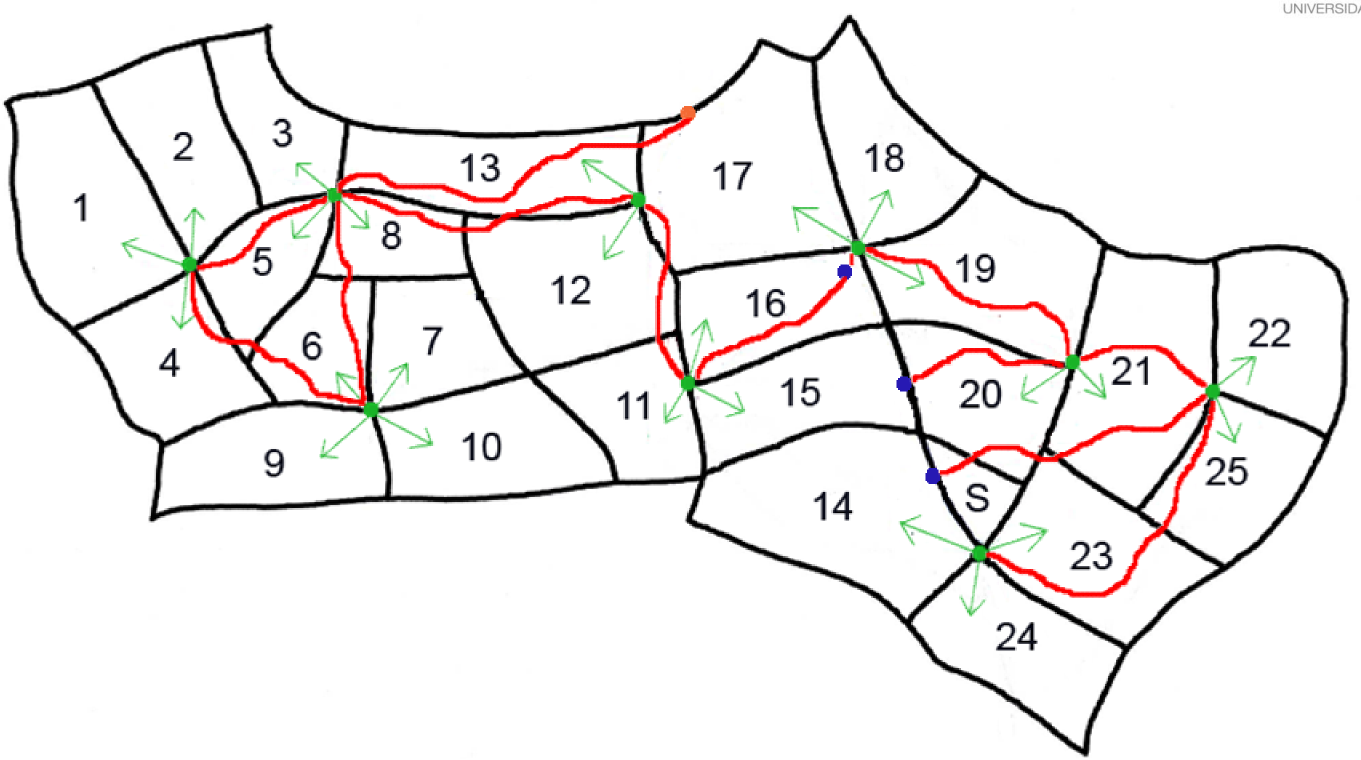
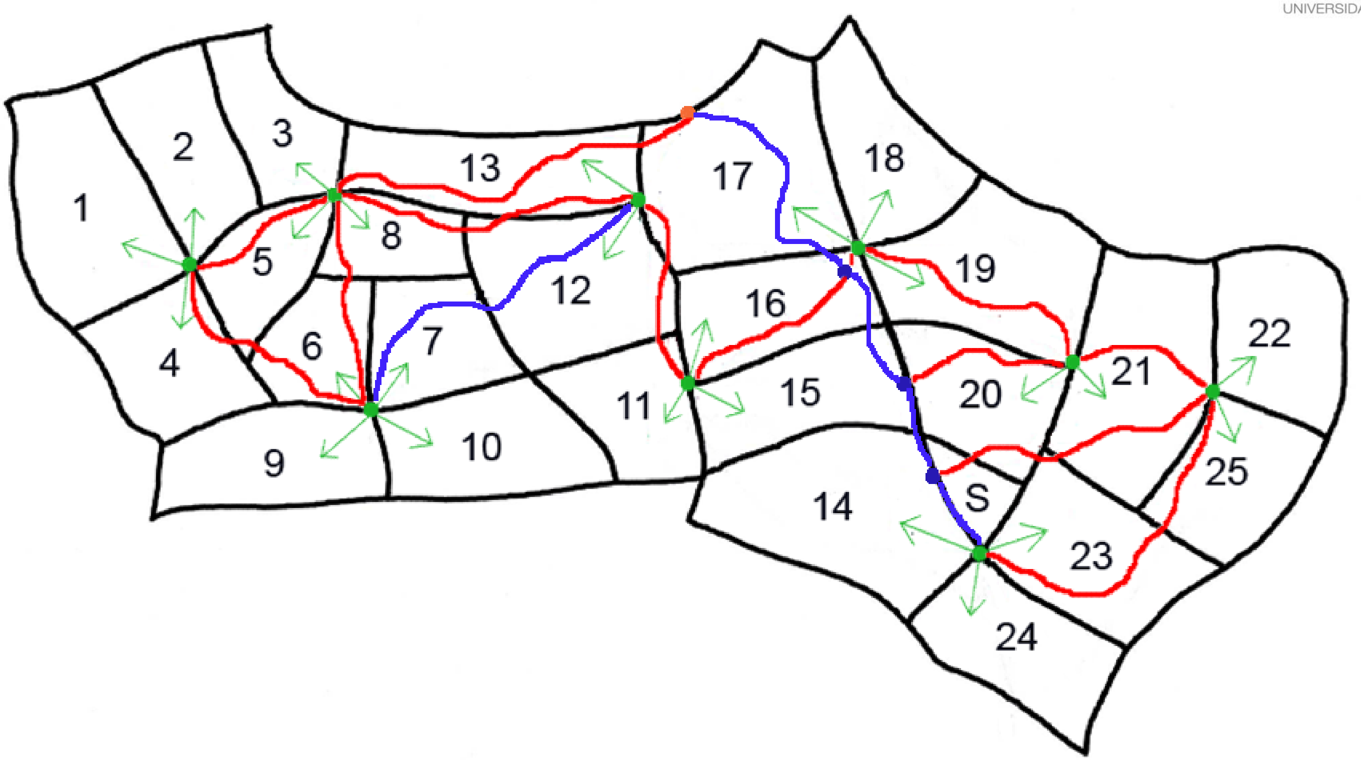


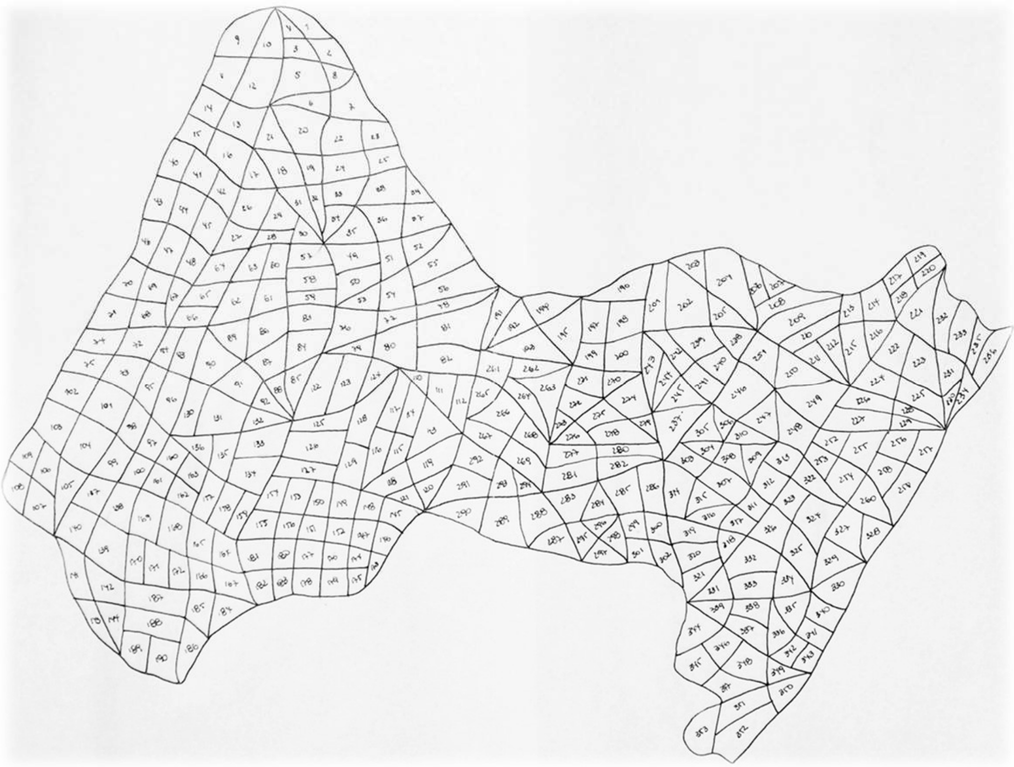
Case Study I



Case Study I



Main Problem

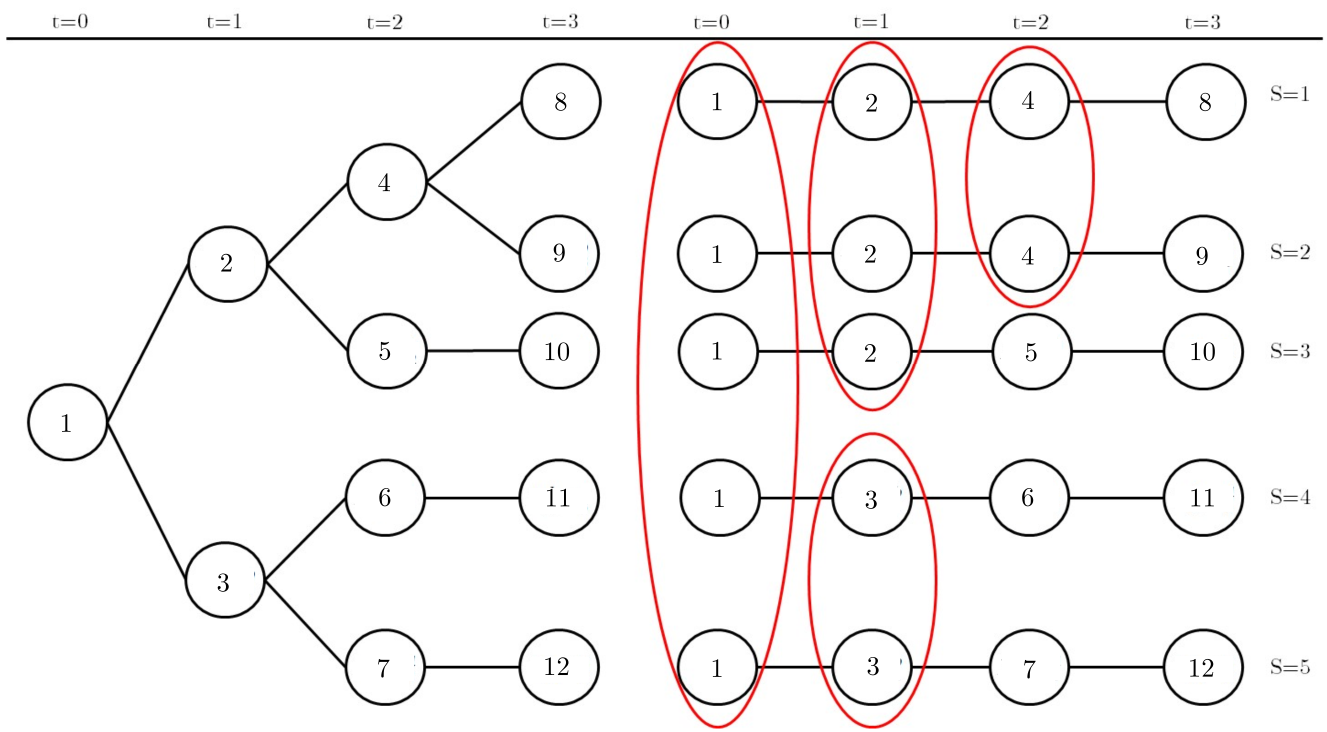


118 Units
290 Roads
3 S. Yards
3 Products
7 Exits
Upgrade

Previous Works

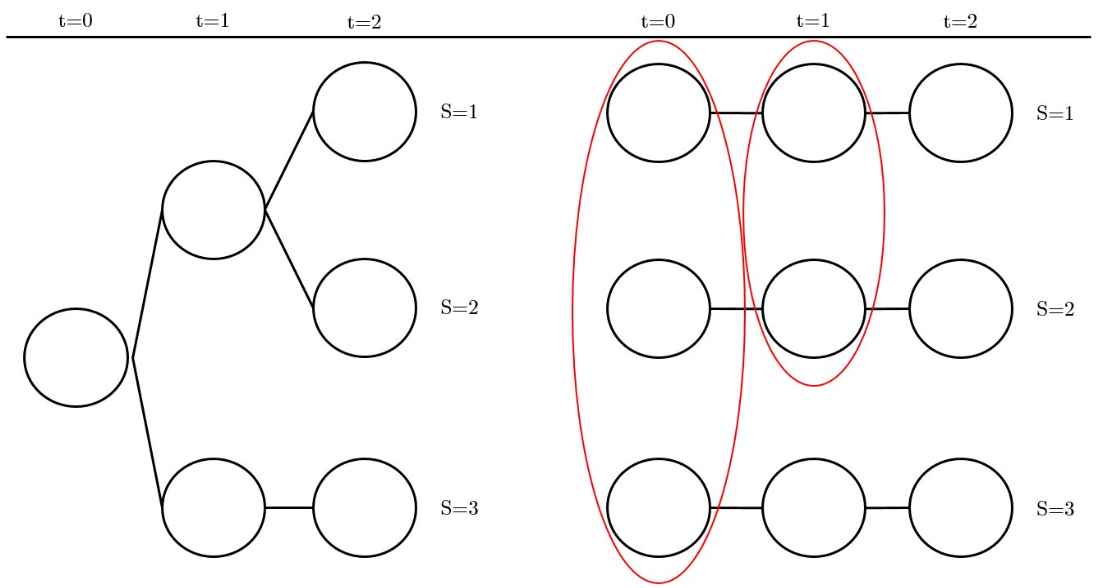
1. Andalaft et. al (2003)
   - Deterministic (17 forests)
   - Lagrangian Relaxation in demand constraints
2. Alonso-Ayuso et. al (2011)
   - Stochastic (prices)
   - 16 scenarios
3. Badilla et. al (2014)
   - Simplified model (1 forest)
   - Price uncertainty (Up to 324 scenarios)

Scenario Trees



Non-anticipativity

"If two scenarios are identical until some stage, then, the decisions taken in those scenarios, until that stage, must be the same".



Extended Formulation

1. Classic Model: Indexed Variables per Scenario
   - More complex constraints
   - Big Problem

2. Explicit Non-anticipativity constraints
   - Exponential growth due to scenarios
   - Difficult to solve

Progressive Hedging

1. Decomposition Algorithm
   - Per Scenarios (horizontal).
   - Accurate with linear variables, heuristic in MIP.
   - Easy to exploit parallel programming.

2. Implicit Non-anticipativity
   - Penalty term in the objective function, based on Lagrangian Relaxation.

Progressive Hedging

$$x_s = x_{s'} \; \forall s, s' \in \mathcal{S} \\ = \mathbb{E}(x_s)$$

$$\Rightarrow \sum_{s \in \mathcal{S}} p_s \{ \min_{x_s \in Q_s} f_s(x_s) + w_s x_s + \frac{\rho}{2}(x_s - \bar{x})^2 \}$$

Pseudo-Code

1 - Solve each scenario $\min_{x_s \in Q_s} f_s(x_s)$

2 - Compute the solution per node $\bar{x} = \sum_{\mathcal{N}_t : s \in \mathcal{N}_t} p_s x_s$

3 - If the solutions are similar enough, STOP $||x - \bar{x}|| < \epsilon$

4 - Update the penalty term $w = \rho(x - \bar{x}) + w_{-1}$

5 - Solve each penalized scenario $\min_{x_s \in Q_s} f_s(x_s) + w_s x_s + \frac{\rho}{2}||x_s - \bar{x}||^2$

6 - Go to 2

Objectives

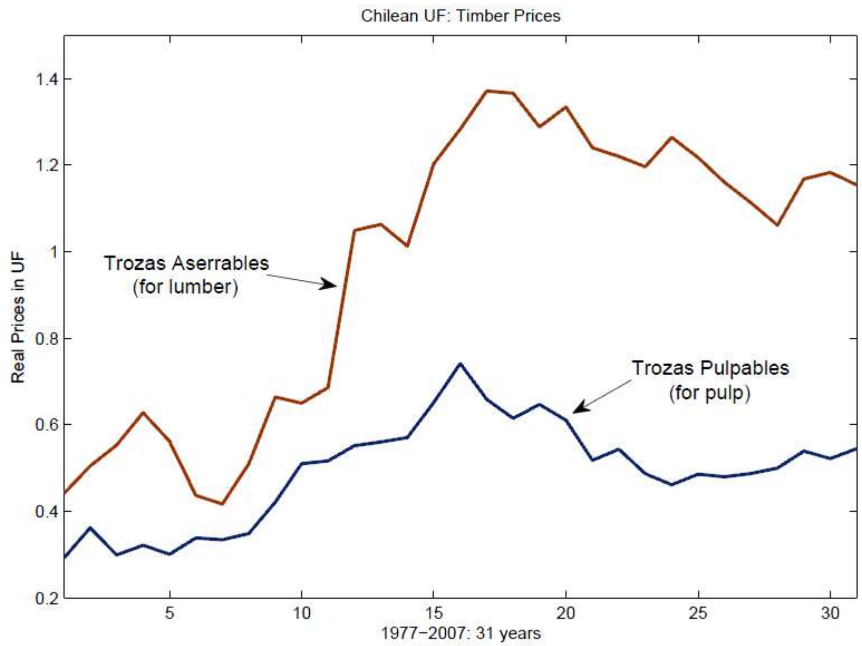
1. General
   - Obtain the optimal plan for a forestry problem including price uncertainty.
2. Specifics
   - Develop a mathematical model to generate price scenarios.
   - Solve and compare different optimization approaches.

First Challenge

- How to generate scenarios, with probabilities.
- One possibility: By logic, experience.
- More rigorous

  Simulate parameters as a random walk with convergence to a mean value, use historical statistics.

Scenario Generation



$$p(t) = v\left(1 - e^{-\mu t}\right) + p_0 \exp\left[(-\mu - \tfrac{1}{2}\sigma^2)t + \sigma w(t)\right].$$

- Rigorous Mathematical model based on price history, using last 20 years data.
- Modeled by stochastic movement with convergence to a mean value, with Weiner Process properties.

Scenario Generation

Prices are modeled by a stochastic differential equation where:

$p_0 =$ Current Price

$\mu =$ Drift (rate)

$v =$ Mean (distant reversion term)

$$dp(t) = \mu(v - p(t))dt + \sigma dw(t)p(t)$$
$$p(0) = p_0$$
$$t \geq 0$$

The approximate solution is:

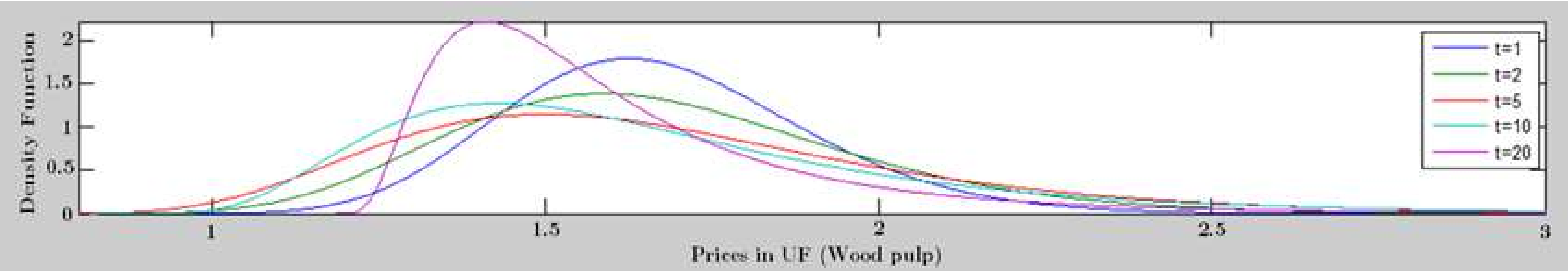
$$p(t) = v(1 - e^{-\mu t}) + p_0 \exp\left[(-\mu - \tfrac{1}{2}\sigma^2)t + \sigma w(t)\right]$$

Where the Expected value and Variance are:

$$E\{p(t)\} = v + (p_0 - v)e^{-\mu t}$$
$$\mathrm{var}(p(t)) = (p_0 e^{-\mu t})^2 (e^{\sigma^2 t} - 1)$$

Scenario Generation

Density Function

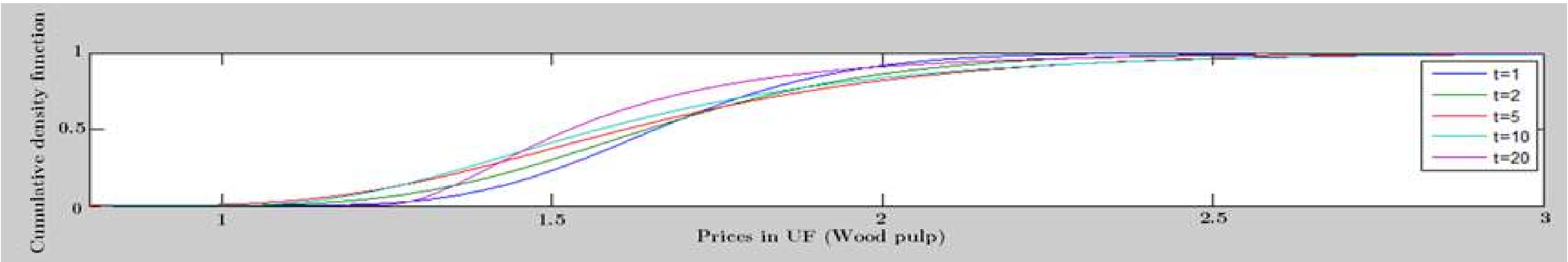


Each period is associated with a particular density function (PDF).

Based on this curves, it is possible to compute the cumulative distribution function (CDF).

Scenario Generation

Cumulative Density Function

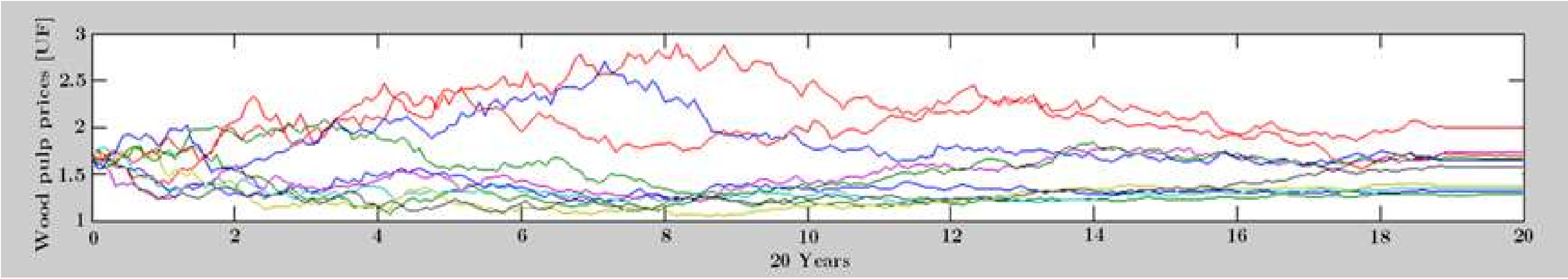


Different discretization of the CDF can be used to generate diferent number of scenarios per period (children nodes).

Once the discretization is made, the conditional expected value for the price in a particular interval is computed, obtaining the value for the new node.

Scenario Generation

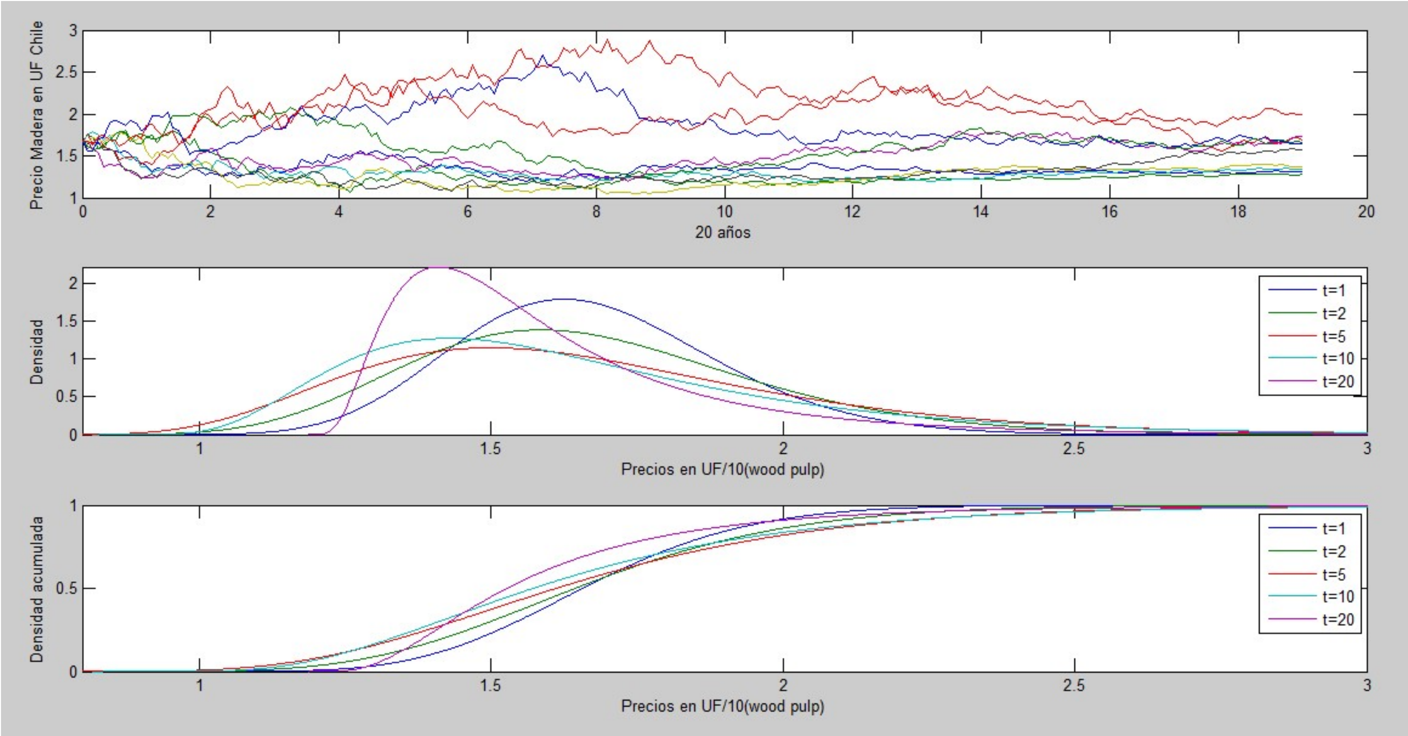
Scenarios



The scenarios are obtained for a particular planning horizon (20 in the example).

Black Swans are included.

Scenario Generation

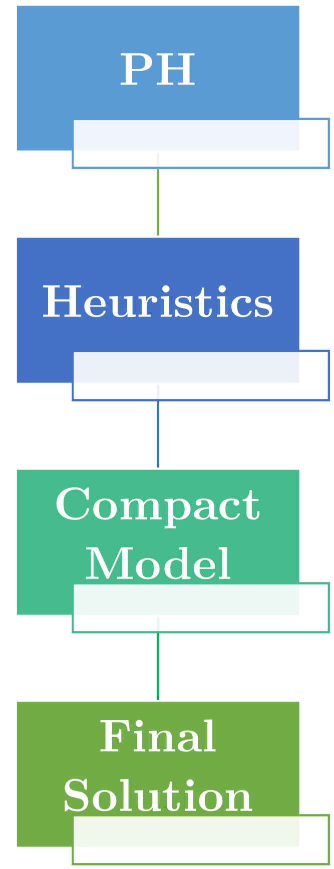


Scenario Generation

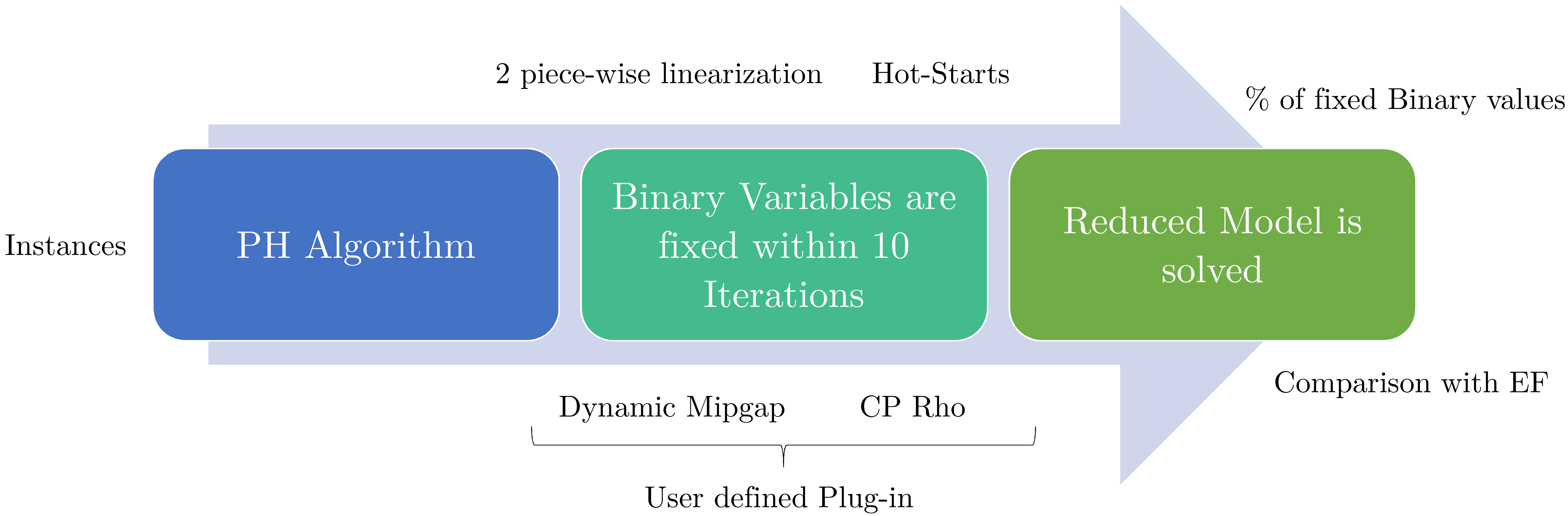
- Each scenario is modeled by the Stochastic movement, using the probability distribution function.
- Scenario values are obtained from the cumulative distribution function (CDF) using the conditional expected value of the distribution, for a particular interval of values.
- Rigorous, flexible and efficient way to generate scenarios.
- Same methodology can be applied to other uncertainties.

Improvements

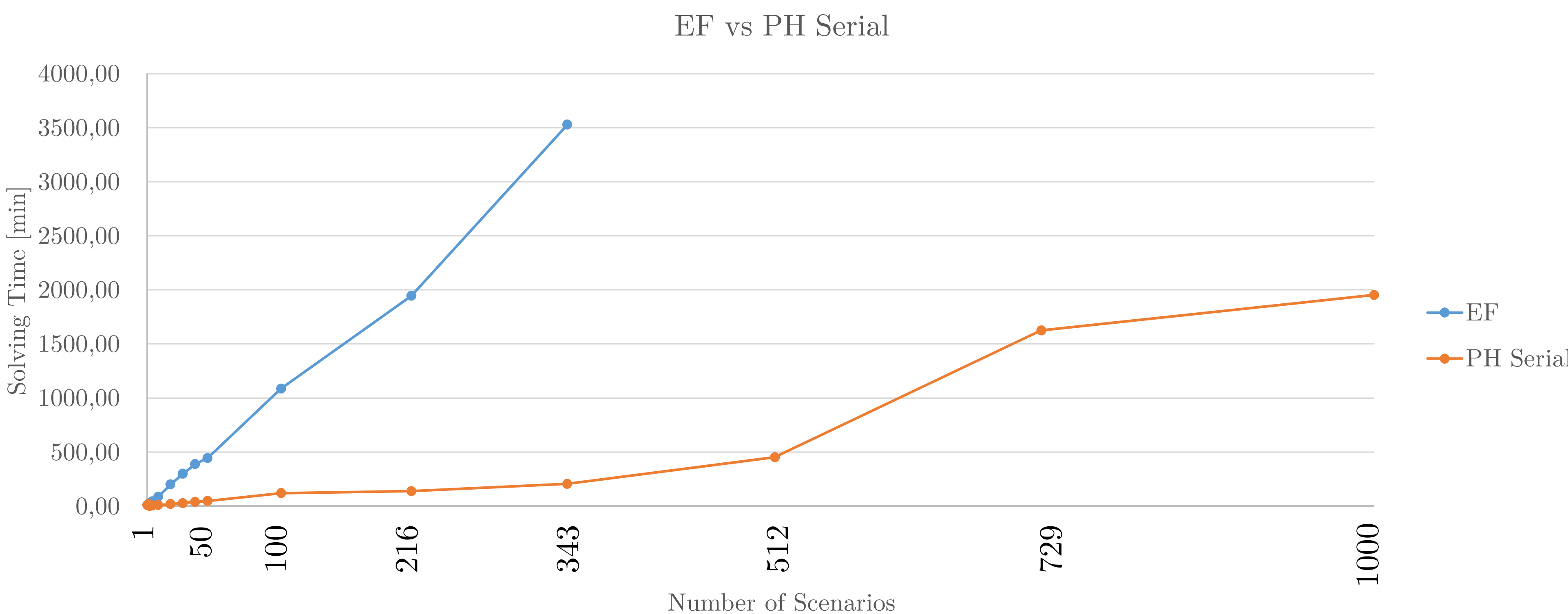
1. Strengthenings & Liftings
2. Hot-starts & fixed variables
3. Dynamic Gap
4. Penalization parameter PH
    - Linearization of quad term.
    - Dynamic rho
5. Bundling & Clustering



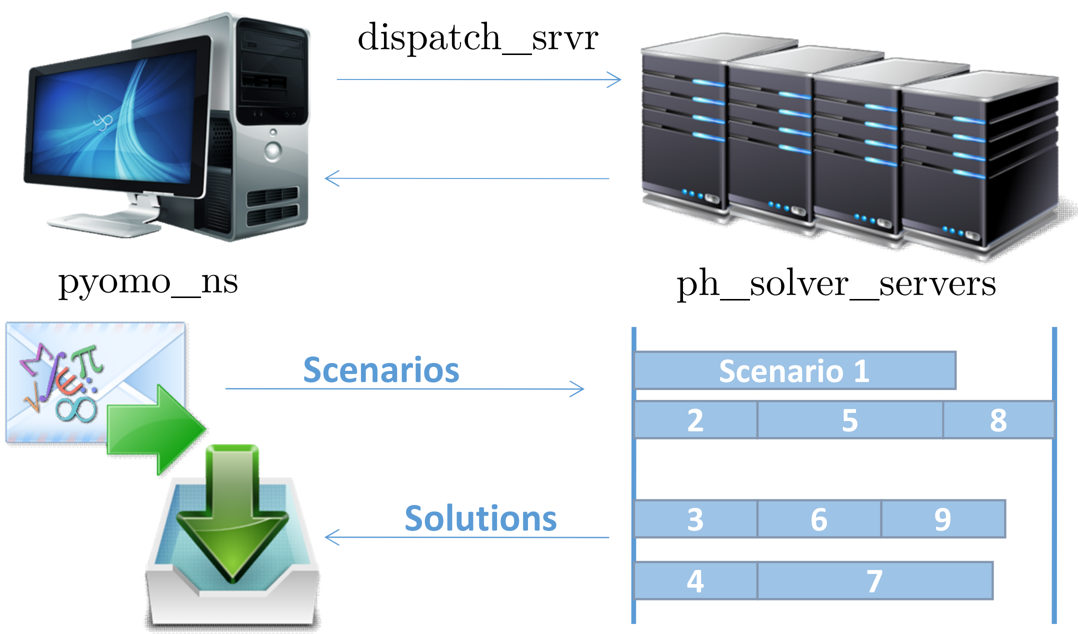
Methodology



Results: Serial



Parallel programming



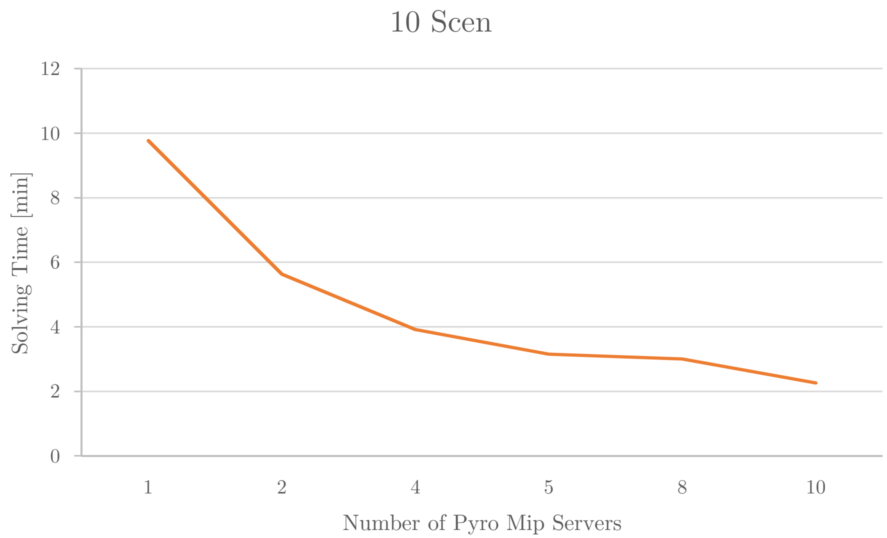
Results: Parallel

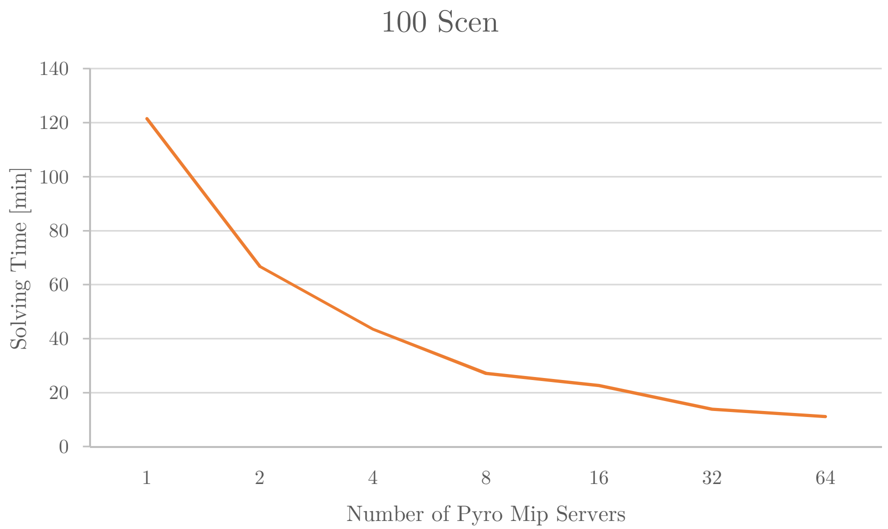
Scenarios	PH Serial	PH Parallel
3	2,63	1,7
5	4,83	1,65
10	9,78	2,26
20	19,91	3,03
30	28,12	3,33
40	40,13	4,65
50	49,27	6,66

Scenarios	PH Serial	PH Parallel
100	121,54	11,15
216	139,13	21,77
343	206,87	31,78
512	452,47	56,88
729	1625,59	108,5
1000	1954,48	287,27

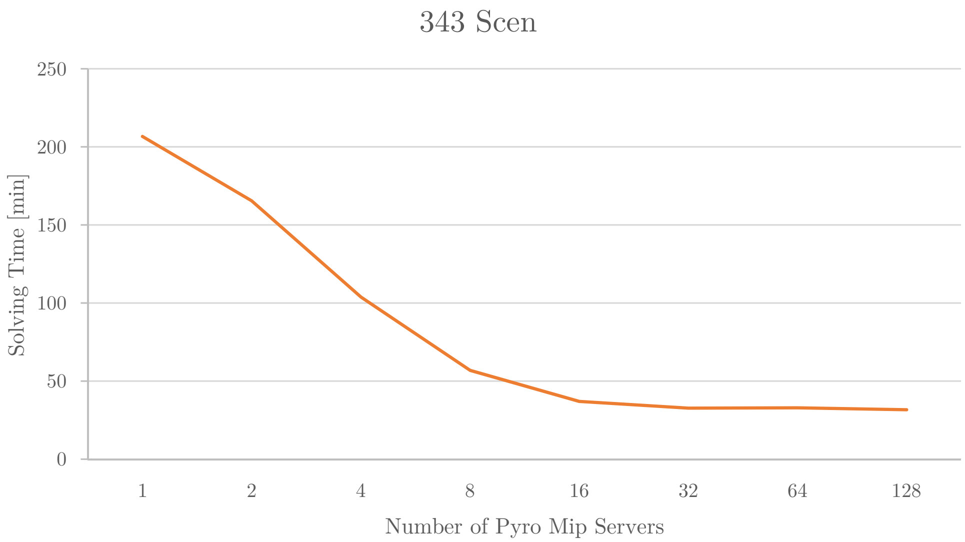
Strengthenings → Hot-Starts → Fixed Variables → Linearization (PH)

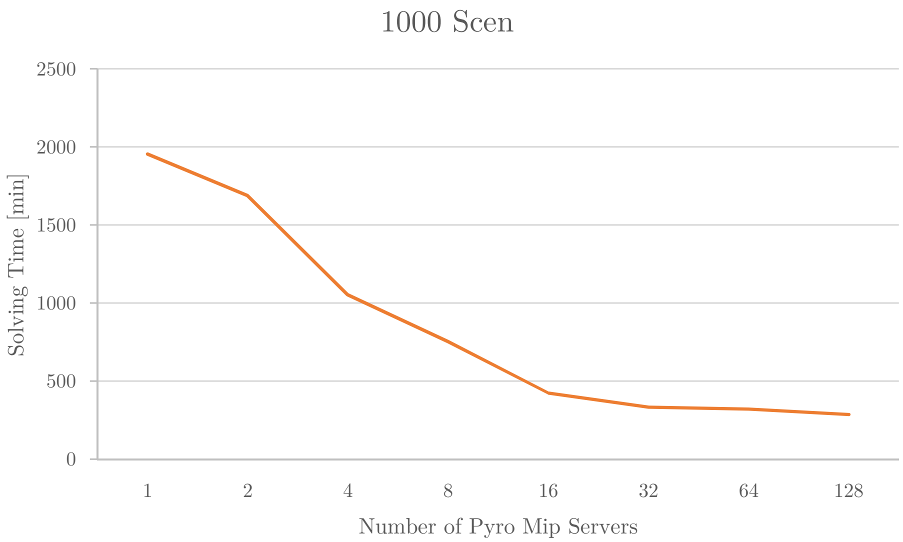
Results: Parallel





Results: Parallel





Conclusions

1. A consistent model for generating price scenarios was implemented, with a strong theoretical basis.

2. PH formulation outperforms the extended model significantly, with a greater performance when the instance grows.

3. The parallel approach was very effective, obtaining the best performance.

Future Work

1. Solve a % of the total number of scenarios and/or delete the most difficult ones and compare the solutions and solving time.

2. Include more uncertainties: growth, climate change, fire, etc.

3. Use of asynchronous parallel optimization.

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